SECTION I, Part A

Time - 60 Minutes

Number of questions - 30

A CALCULATOR MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

1. $\int e^{2x}\cos x\ dx =$

(A) $\dfrac{e^{2x}(\sin x+\cos x)}{3}+C$

(B) $\dfrac{e^{2x}(\sin x-2\cos x)}{3}+C$

(C) $\dfrac{e^{2x}(\sin x+2\cos x)}{5}+C$

(D) $\dfrac{e^{2x}(\sin x-\cos x)}{5}+C$

2. If $f(x)=\cos(e^{-\tan x})$, then $f'(x)=$

(A) $-\sin(e^{-\tan x})$

(B) $e^{-\tan x}\sin x\sec^2(e^{-\tan x})$

(C) $e^{-\tan x}\sin(e^{-\tan x})$

(D) $e^{-\tan x}\sin(e^{-\tan x})\sec^2 x$

3. If $F(x) = \int_0^x t^3 + 2\ dt$, then $F(2) =$

(A) 4

(B) 8

(C) 10

(D) 12

4. If $f(x) = 2\cos^2 x + \tan x$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $4 - \sqrt{3}$

(B) $\frac{4 - 3\sqrt{3}}{3}$

(C) $\frac{4 + 3\sqrt{3}}{3}$

(D) $4 + \sqrt{3}$

5. At which of the following points is the graph of $f(x) = x^5 - 10x^3$ increasing and concave up?

(A) $(-\infty, -\sqrt{6})$

(B) $(-\sqrt{3}, 0)$

(C) $(0, \sqrt{6})$

(D) $(\sqrt{6}, \infty)$

6. Which of the following are antiderivatives of $f(x) = \sin^2 x \cos x + 2\cos x \sin x$?

I. $F(x) = \dfrac{\sin^3 x}{3} + \sin^2 x$

II. $F(x) = \dfrac{2 + \sin^3 x}{3} - \cos^2 x$

III. $F(x) = \dfrac{\cos^3 x}{3} - \dfrac{\cos^2 x}{2}$

(A) I only

(B) II only

(C) II and III

(D) I and II

7. If $x = \sin 2t$ and $y = \tan 2t$, then $\frac{dy}{dx} =$

(A) $\frac{\sec 2t}{\sin 2t}$

(B) $\frac{1}{\cos^2 2t}$

(C) $\frac{1}{\cos^3 2t}$

(D) $2\cos^2 2t$

8. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{(2x)^n}{n!}$.

What is the value of $f(5)$?

(A) e^{10}

(B) $\frac{1}{e^{10}}$

(C) $\ln 3$

(D) $\cos 10$

9. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n(x-5)^{2n}}{2^n}$?

(A) $\sqrt{2}$

(B) 2

(C) 5

(D) 7

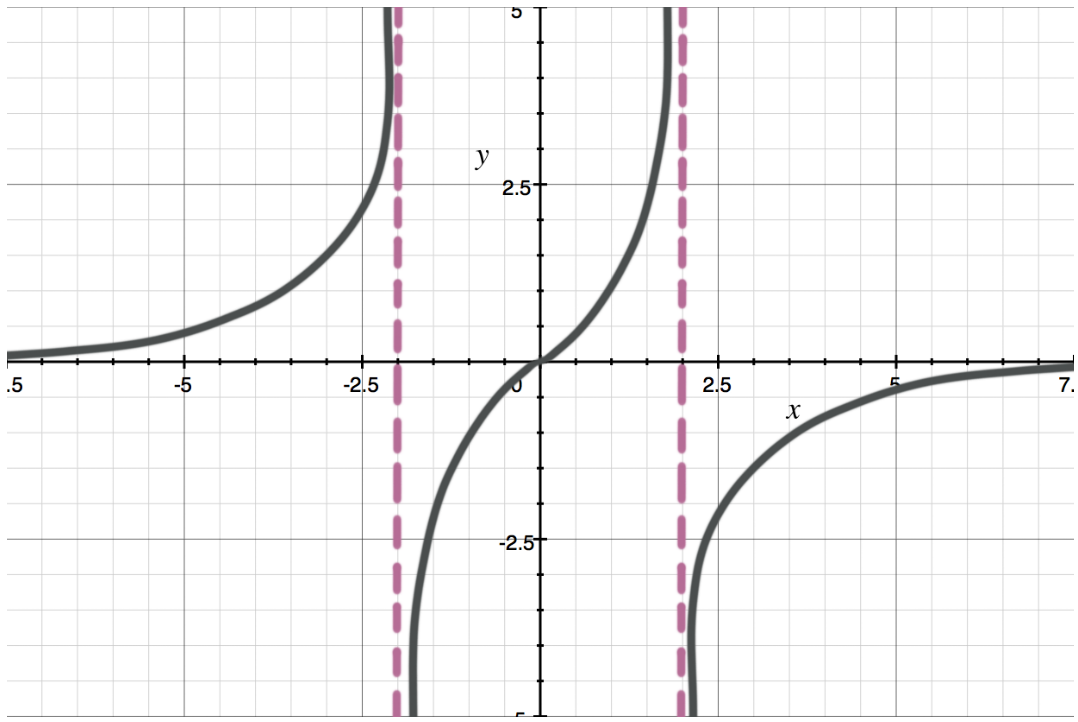
10. What is the slope of the line tangent to the polar curve $r = 4 + 5\cos\theta$ at $\theta = \frac{\pi}{2}$?

(A) $-\frac{4}{5}$

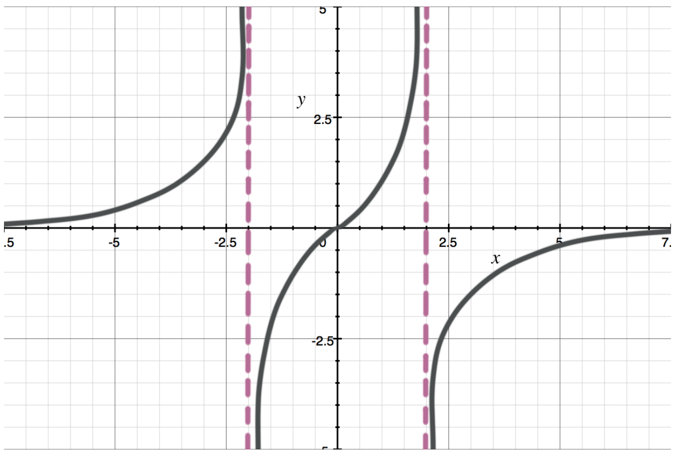
(B) 0

(C) 1

(D) $\frac{5}{4}$



11. The graph of the derivative of f is given above. Which of these graphs could be the function f?

(A)


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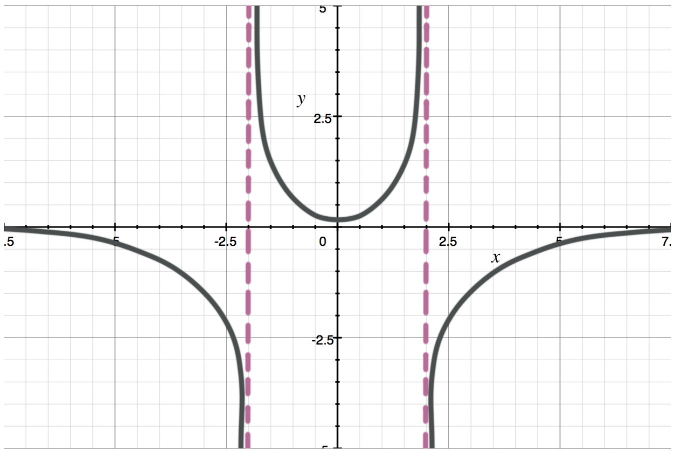
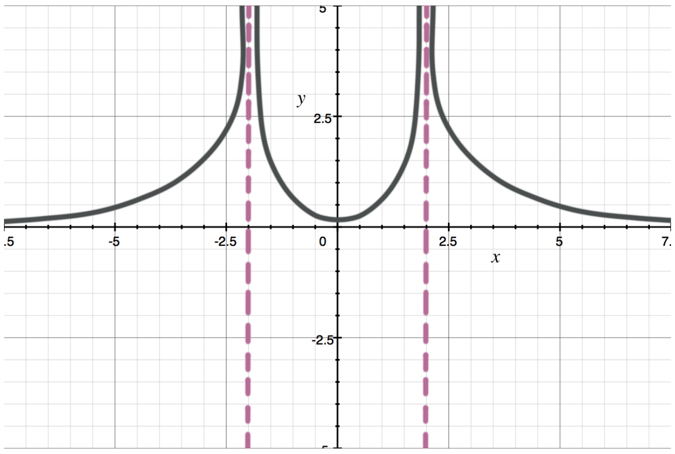
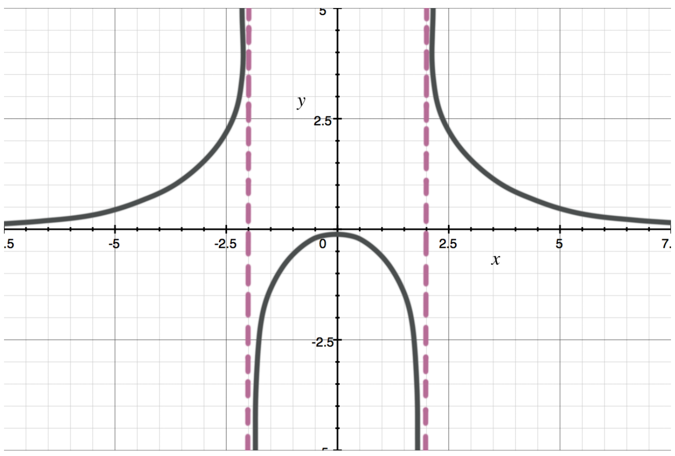


(C) 

(D) 

12. The function f is given by $f(x) = x^4 - 10x^3 + 25x^2 - 36$. All of these statements are true, except

(A) -1 and 2 are zeros of f.

(B) $f'(0) = 0$

(C) $(0, -36)$ is a local maximum of f.

(D) $(5, -36)$ is a local minimum of f.

13. Let f be a function defined for all real numbers x. If $f'(x)=\dfrac{|16-x^2|}{x-4}$, then f is decreasing on which of the following intervals?

(A) $(-\infty,4)$

(B) $(-4,4)$

(C) $(-4,\infty)$

(D) $(4,\infty)$

14. Let f and g be twice differentiable functions such that $g'(x)\le 0$ for all x in the domain of f. If $h(x)=g(f'(x))$ and $h'(2)=-3$, then which of the following is true at $x=2$?

(A) h is concave down.

(B) f is concave up.

(C) g is concave up.

(D) f is increasing.

15. For what values of f does the curve given by the parametric equation $x = t^3 - 3t^2 + 3t + 5$ and $y = t^2 - 4t + 5$ have a horizontal tangent?

(A) 0 only

(B) 0 and 1 only

(C) 1 and 2 only

(D) 2 only

16. What is the area of the region bounded by the curves $y = x^2 - x - 12$ and $y = -x^3$ from $x = 0$ to $x = 2$?

(A) $\frac{46}{3}$

(B) $\frac{58}{3}$

(C) $\frac{70}{3}$

(D) $\frac{98}{3}$

17. Determine $\frac{dy}{dx}$ for the curve defined by $3x^2 + y^3 = 3xy$.

(A) $\frac{y - x^2}{y^2 - x}$

(B) $\frac{2x - y}{y^2 - x}$

(C) $\frac{2x}{y - y^2}$

(D) $\frac{y - 2x}{y^2 - x}$

18. For $x > 0$, the power series $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n)!} + \ldots$ converges to which of the following?

(A) $\cos x$

(B) $\sin x$

(C) $x \cos x$

(D) xe^x

19. For what values of x does the series $1+\frac{2^x}{2^2}+\frac{3^x}{3^2}+\frac{4^x}{4^2}+\ldots+\frac{n^x}{n^2}+\ldots$ converge?

(A) $x<-1$

(B) $x\geq -1$

(C) $x<1$

(D) All values of x

20. What is the average value of $y=\frac{3}{x}$ over $[1,e]$?

(A) 1

(B) 3

(C) $\frac{3}{e-1}$

(D) $\frac{1}{e-1}$

21. $\lim_{x\to\infty} \frac{x^2 - 4x + 5}{x^2 - x^3 + 8} =$

(A) -1

(B) 0

(C) 1

(D) Does not exist.

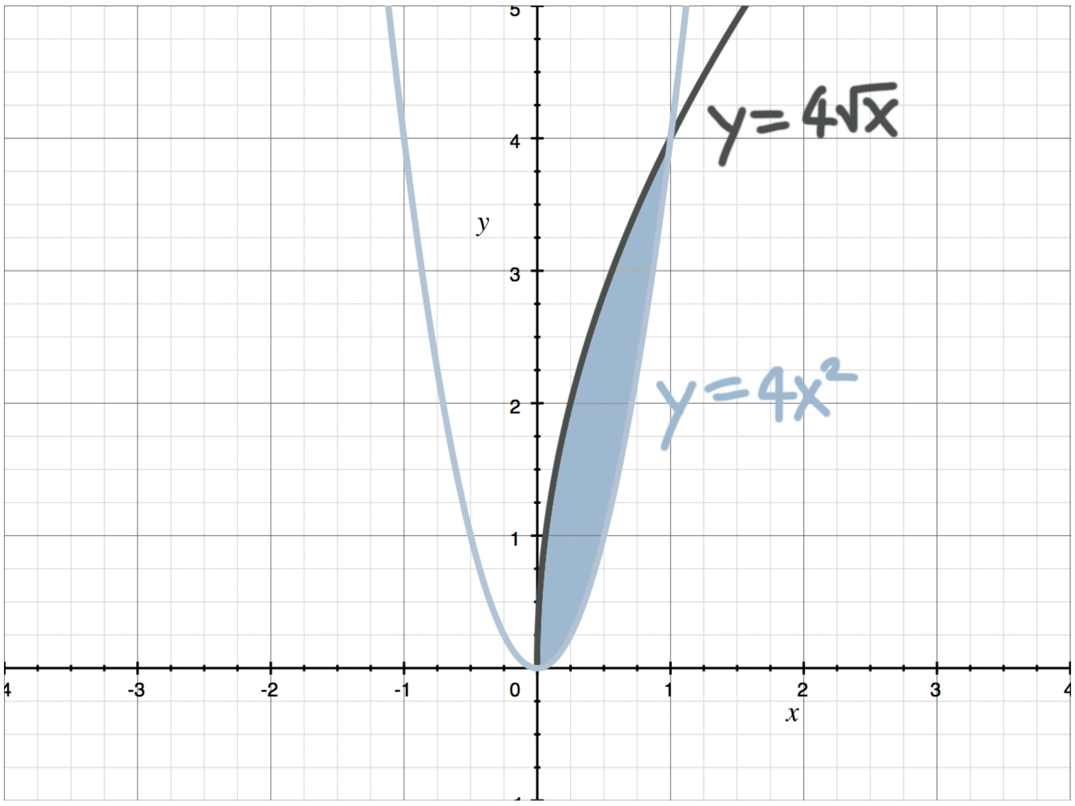
22. The graph of $f(x) = \frac{2 + x}{x^2 - 4}$ is concave up over which of the following interval(s)?

(A) $(-\infty, 2)$

(B) $(-\infty, -2)$ and $(-2, 2)$

(C) $(2, \infty)$

(D) $(-\infty, \infty)$



23. The area of the shaded region in the diagram above is equivalent to

(A) $\int_0^4 4x^2 - 4\sqrt{x}\ dx$

(B) $\pi \int_0^4 16x^4 - 16x\ dx$

(C) $\int_0^1 4\sqrt{x} - 4x^2\ dx$

(D) $2\pi \int_0^1 \sqrt{x} - x^2\ dx$

24. $\lim_{h\to 0}\dfrac{3\cos 2\left(\frac{\pi}{4}+h\right)-3\cos\frac{\pi}{2}}{h}=$

(A) -6

(B) -3

(C) 0

(D) 2

25. $\int_0^{\frac{\pi}{2}}\sin^3 x\cos x\ dx=$

(A) $-\dfrac{1}{4}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{\pi^4}{64}$

(D) $\dfrac{\pi^4}{16}$

26. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = 2t^3 - 3t^2$ and $y(t) = 6t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(0,1)$

(B) $(1,1)$

(C) $(-1,3)$

(D) $(3,1)$

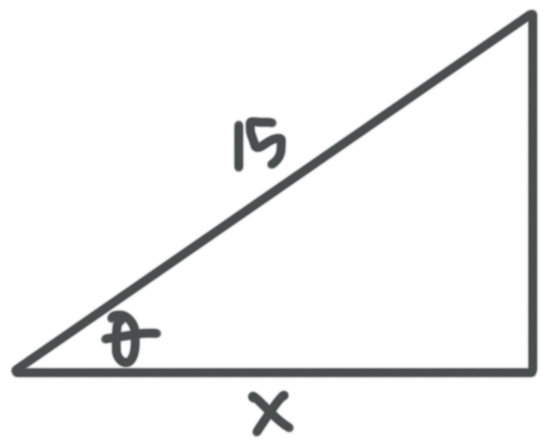
27. If $f(x) = \begin{cases} 4\ln x^2 & 0 < x < 2 \\ x^2 \ln 4 & x \geq 2 \end{cases}$, then $\lim_{x \to 2} f(x) =$

(A) 4

(B) $\ln 4$

(C) $4 \ln 4$

(D) The limit does not exist.



28. In the triangle shown above, θ is at decreasing at a constant rate of 5 radians per minute. What is the rate of change of x, in units per minute, when $x=9$ units?

(A) -60

(B) 36

(C) 45

(D) 60

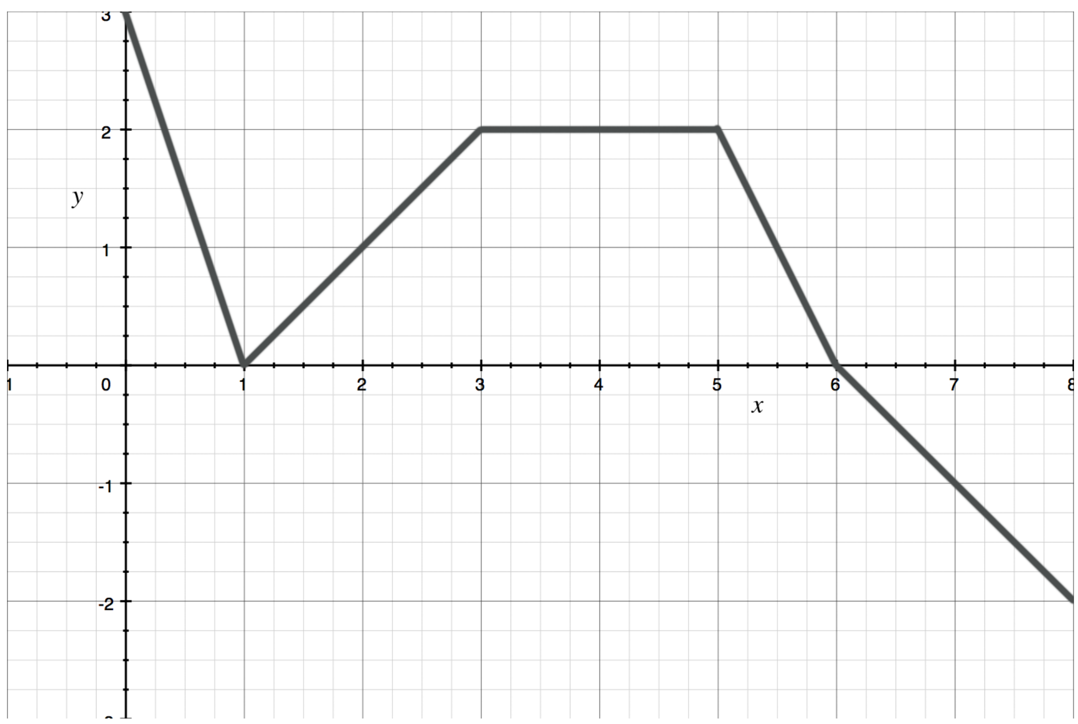
29. If $f(x)=\dfrac{\cos x^3}{e^{2x^2}}$, then $f'(x)=$

(A) $\dfrac{-3x\sin x^3}{4e^{2x^2}}$

(B) $\dfrac{3x^2\sin x-4x\cos x^3}{e^{2x^2}}$

(C) $\dfrac{3x^2\sin x^3+4x\cos x^3}{e^{4x^2}}$

(D) $\dfrac{-3x^2\sin x^3-4x\cos x^3}{e^{2x^2}}$



30. The graph of the derivative of f is shown above. If $f(0) = 0$, then which of the following statements is true?

(A) $f(8) < f(6) < f(1) < f(3)$

(B) $f(8) < f(1) < f(6) < f(3)$

(C) $f(1) < f(3) < f(6) < f(8)$

(D) $f(1) < f(3) < f(8) < f(6)$

END OF PART A, SECTION I

SECTION I, Part B

Time - 45 **Minutes**

Number of questions - 15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

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31. Which of the following sequences converge?

I. $\left\{ \frac{2n}{3-4n} \right\}$

II. $\left\{ \frac{e^{3n}}{2-e^{2n}} \right\}$

III. $\left\{ \frac{e^{3n}}{3n} \right\}$

(A) I only

(B) I and II only

(C) II and III only

(D) I, II, and III

32. $\int \frac{1}{(x-4)(x-2)}\ dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

(B) $\frac{1}{2}\ln|(x-2)(x-4)| + C$

(C) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+6}{(x-2)(x-4)}\right| + C$

33. If g is a differentiable function such that $g(x) > 0$, for all real numbers x, and if $f'(x) = (x^2 + 2x - 8) \cdot g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -4$ and a relative minimum at $x = 2$.

(B) f has a relative maximum at $x = 2$ and a relative minimum at $x = -4$.

(C) f has a relative minimum at $x = 2$ and $x = -4$.

(D) f has a relative maximum at $x = 2$ and $x = -4$.

34. The radius of a circle is increasing at a constant rate of 0.15 centimeters per second. What is the rate of change of the area of the circle, in square centimeters per second?

(A) $0.3\pi C$

(B) $0.15C$

(C) $-0.15C$

(D) $-\dfrac{0.3C}{\pi}$

35. Let $f(x)=\dfrac{|x^2-4|}{x+2}$. Which of these statements is(are) false?

I. f is not continuous at $x=2$.

II. f is differentiable at $x=-2$.

III. f has a local minimum at $x=2$.

(A) I only

(B) I and II only

(C) II and III only

(D) I, II, and III

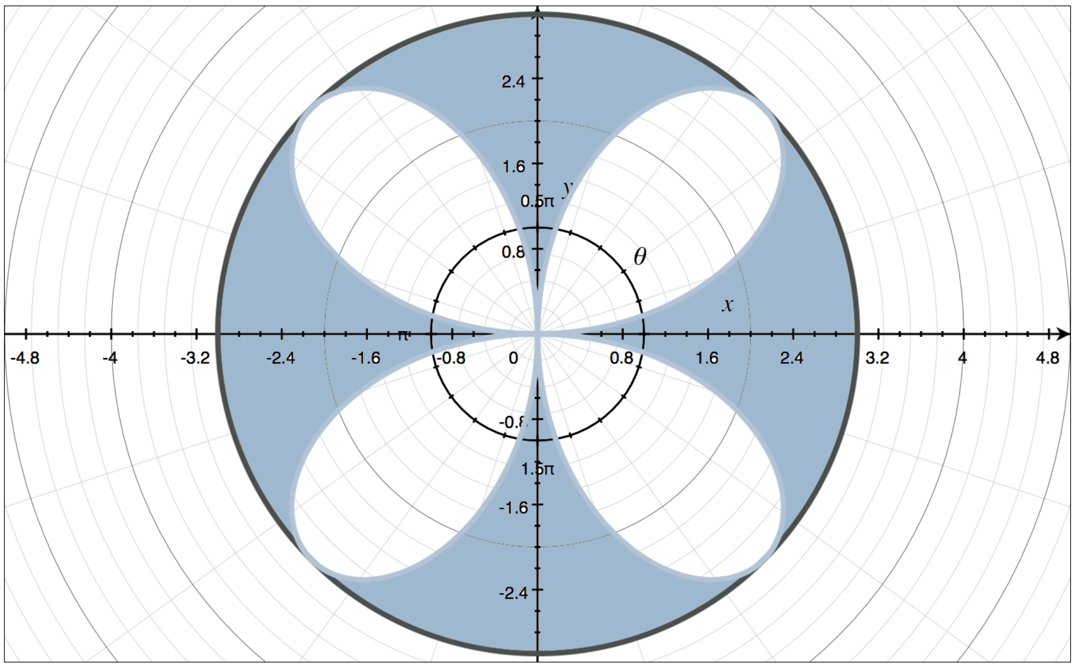
36. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane is given by the parametric equations $x'(t) = \frac{2}{t+3}$ and $y'(t) = 2ke^{kt}$, where k is a positive constant. The line $y = 2t - 5$ is parallel to the line tangent to the path of the particle at the point where $t = 3$. What is the value of k?

(A) 0.117

(B) 0.189

(C) 0.242

(D) 0.378



37. The figure above shows the graphs of the polar curves $r = 3\sin 2\theta$ and $r = 3$. What is the area of the shaded region?

(A) 3.534

(B) 14.137

(C) 24.740

(D) 28.274

38. When the region enclosed by the graphs of $y = 2x$ and $y = 6x - x^2$ is revolved about the y-axis, what is the volume of the resulting solid?

(A) 10.667

(B) 67.021

(C) 134.041

(D) 544.545

39. Let y be the function given by $f(x) = 2e^{3x}$ and let g be the function given by $g(x) = 5x^3 - 2x$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.366

(B) -0.478

(C) -0.251

(D) -0.414

40. For $-1 < x < 1$, if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n\ x^{2n-1}}{2n-1}$, then $f'(x) =$

(A) $\sum_{n=1}^{\infty} (-1)^{n-1} x^{2n-2}$

(B) $\sum_{n=1}^{\infty} x^{2n}$

(C) $\sum_{n=1}^{\infty} (-1)^n x^{2n-2}$

(D) $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ x^{2n}}{2n}$

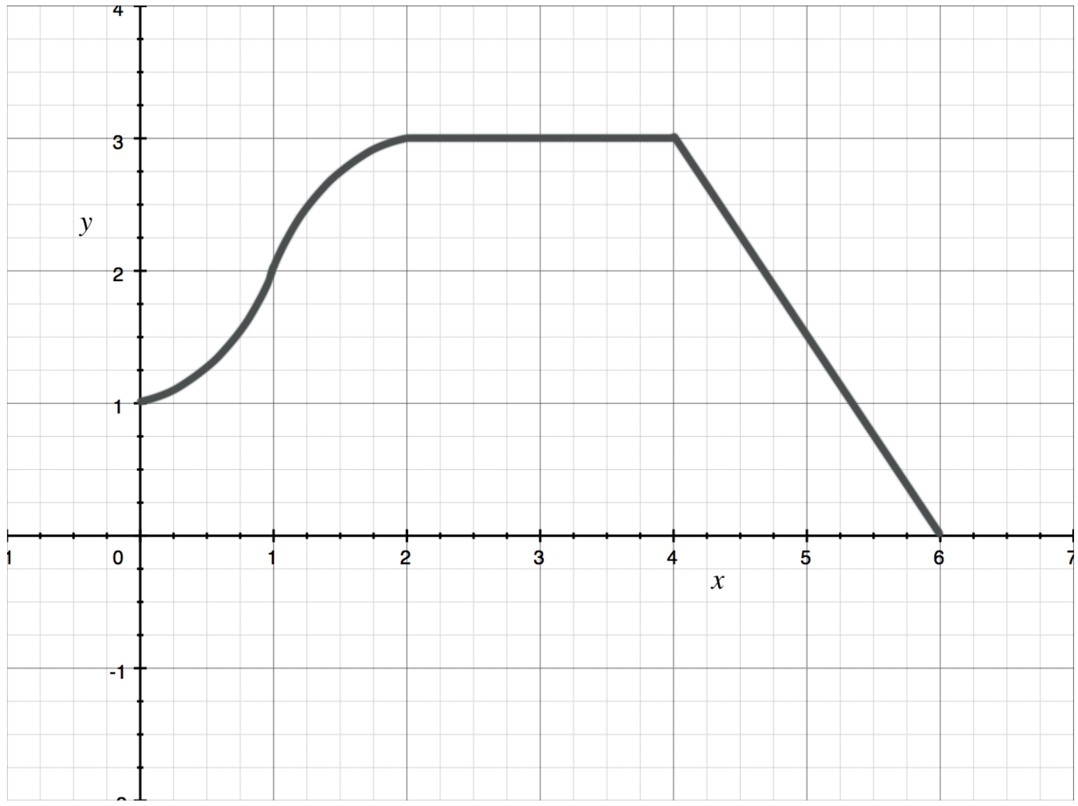
41. The graph of the function $y = 2x^3 + 3x^2 + 5 - 2\sin x$ changes concavity at $x =$

(A) -0.593

(B) -0.430

(C) 0.257

(D) 0.484



42. The graph of f is shown in the figure above. If $\int_0^2 f(x)\,dx = 4.2$ and $F'(x) = f(x)$, then $F(6) - F(0) =$

(A) 1.3

(B) 10.2

(C) 13.2

(D) 16.2

x	0	2	4	6	8	10	12
f(x)	5.0	2.3	3.1	1.0	4.5	6.2	4.6

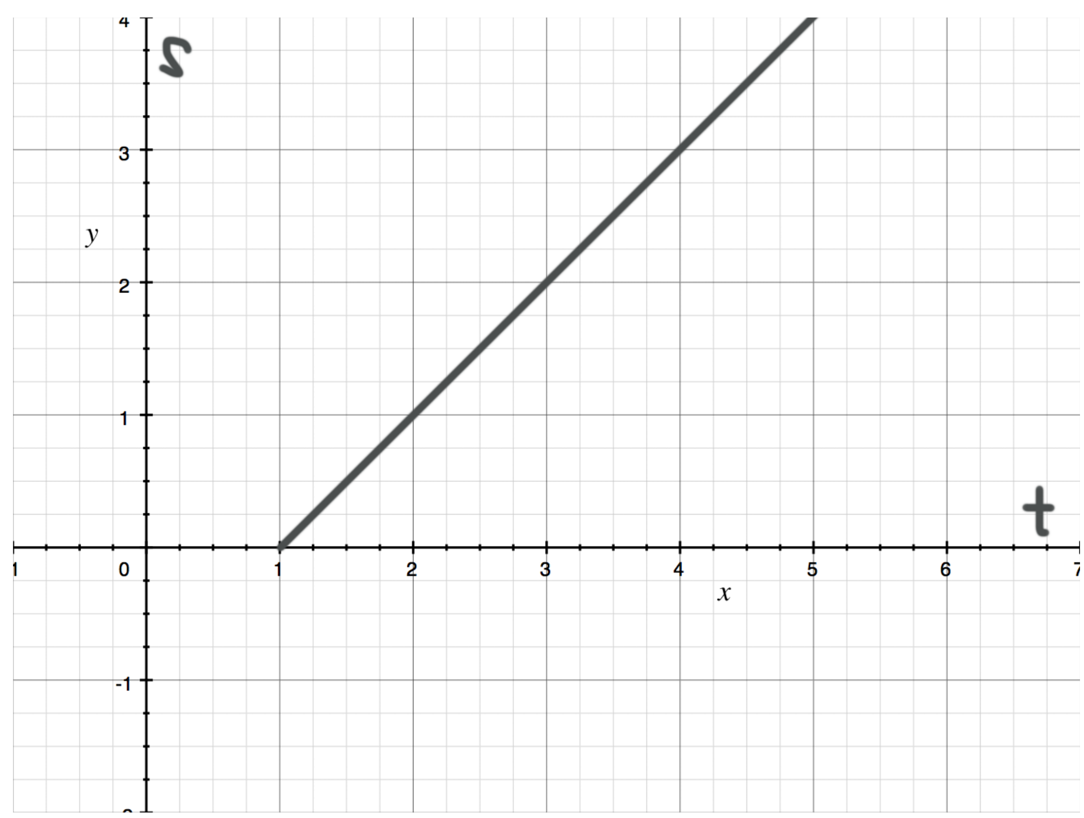
43. A table of values for a continuous function f is shown above. If six equal subintervals of $[0,12]$ are used, which of the following is equivalent to a right-hand Riemann sum approximation for $\int_0^{12} f(x)\, dx$?

(A) 21.7

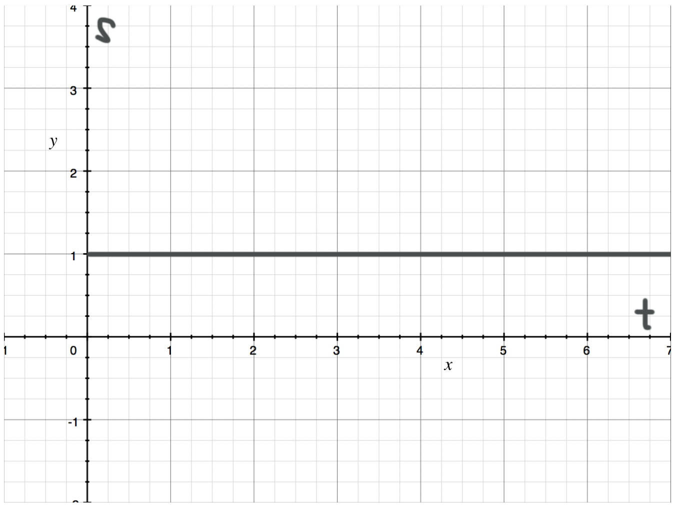
(B) 43.4

(C) 44.2

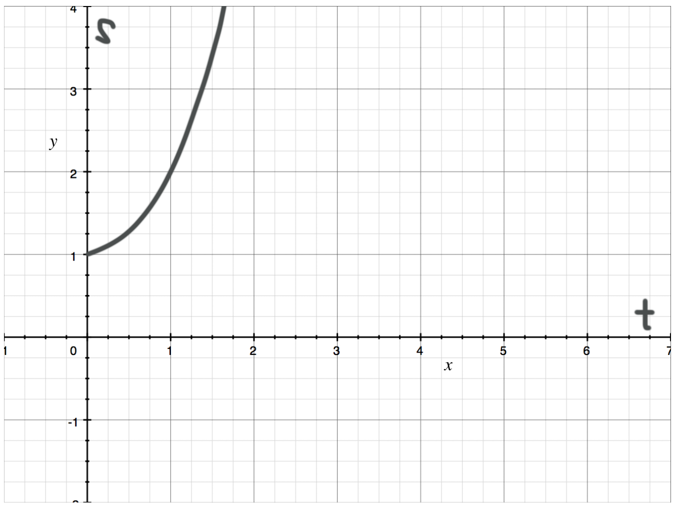
(D) 53.4



44. The graph of the distance $s(t)$ of the particle as a function of time t is shown above. Which of the following could be the graph of the velocity $v(t)$ of the particle?



(A)



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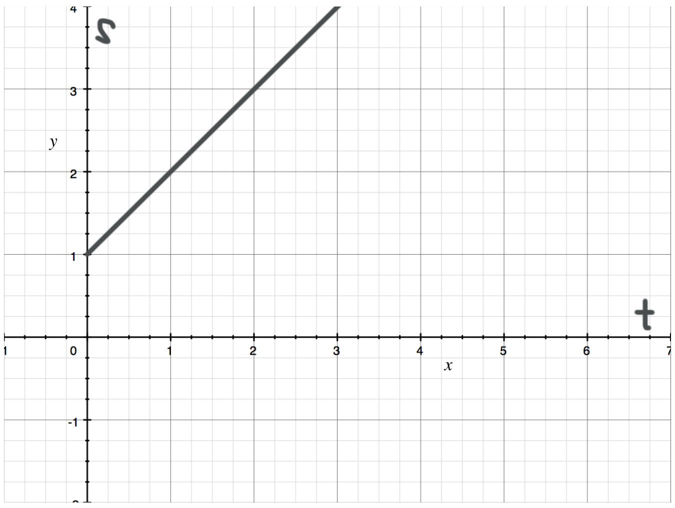
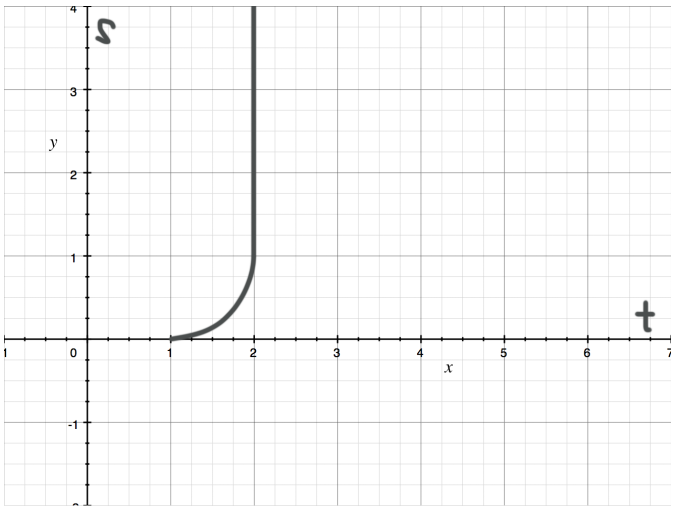


(C)

(D) 

45. If $a \neq 0$, then $\lim_{x \to a} \frac{x^3 - a^3}{x^2 - a^2} =$

(A) $\frac{1}{2}a$

(B) $\frac{3}{2}$

(C) 0

(D) $\frac{3}{2}a$

END OF PART B, SECTION I

SECTION II, PART A

Time - 30 Minutes

Number of problems - 2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.

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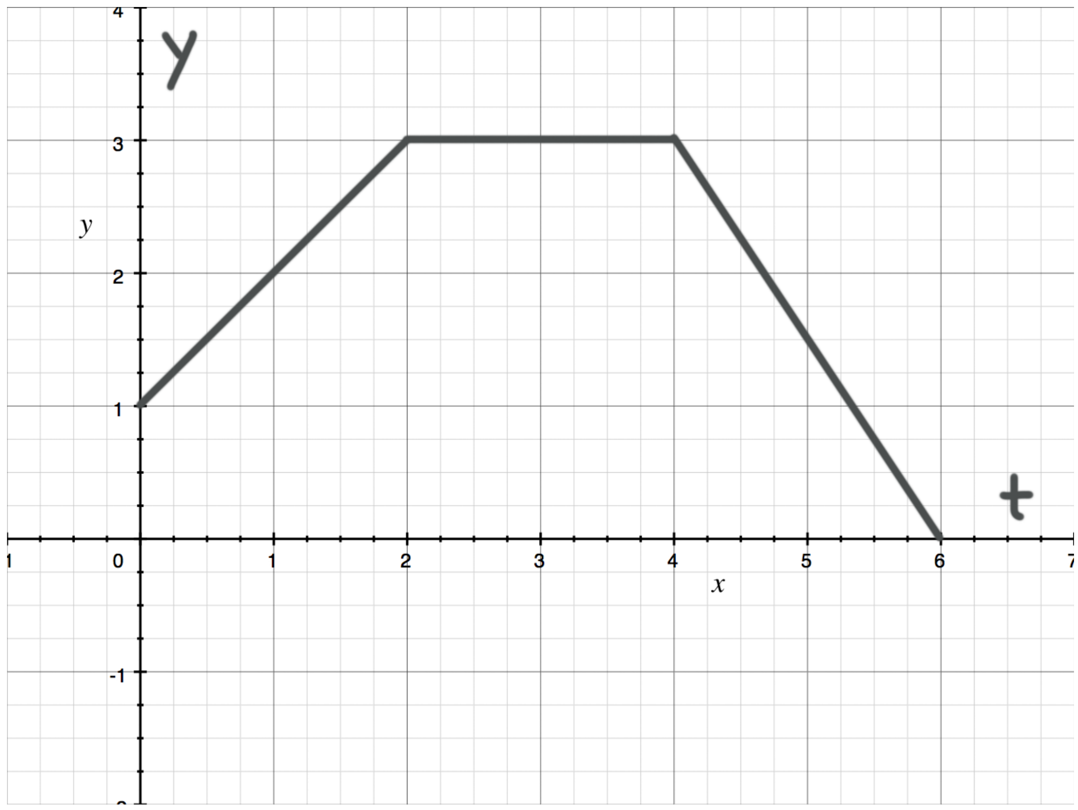
1. A water tank has the shape of an open circular cone. The height of the water tank is 20 cm and the radius of the opening is 10 cm. At height h, the radius of the water tank is given by $r = \frac{1}{10}(h^2 - 6)$, where $0 \leq h \leq 20$.

a. Find the average value of the radius of the water tank.

b. Find the volume of the water tank.

c. Find the rate of change of the volume of water in the water tank with respect to time and when $h = 5$ cm, if water is evaporating so that its depth h is changing at the constant rate of $-\frac{2}{5}$ cm/hr.



2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + 2\sin(2t^2 + 1)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(3,1)$.

a. Find the position of the particle at $t = 1$.

b. Find the slope of the line tangent to the path of the particle at $t = 1$.

c. Find the speed of the particle at $t = 5$.

d. Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

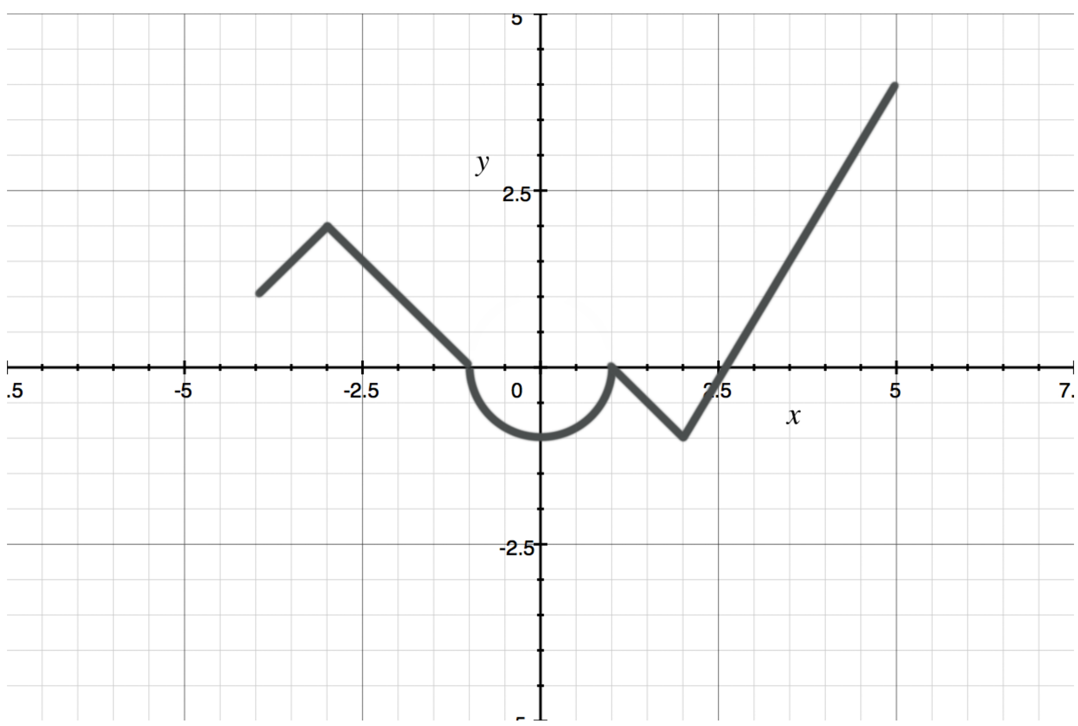
END OF PART A, SECTION II

SECTION II, PART B

Time - 60 Minutes

Number of problems - 4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.



3. The graph of the function f shown above consists of a semicircle and four line segments. Let g be the function given by $g(x)=\int_{-3}^{x} f(t)\ dt$.

a. Find $g(-2)$, $g'(-2)$, and $g''(-2)$.

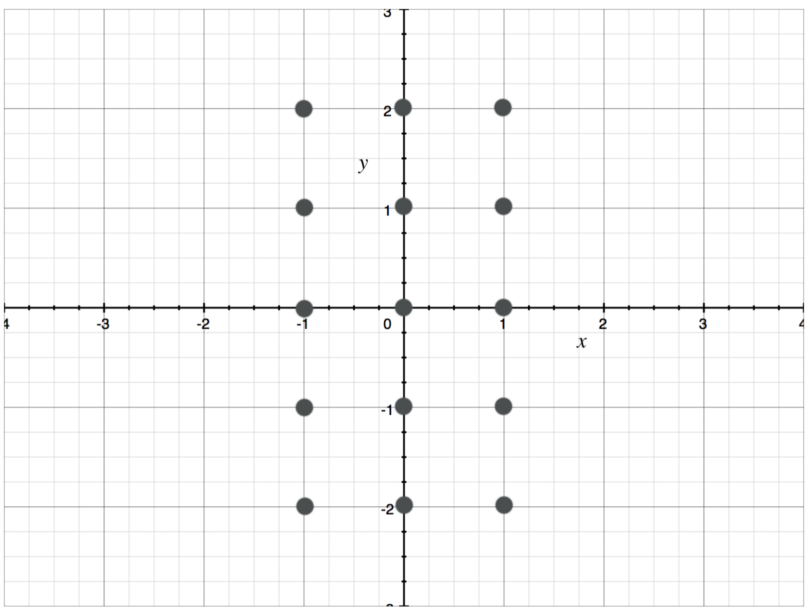
b. Does g have a relative minimum, a relative maximum, or neither at $x=1$? Explain your answer.

c. Find all values of x in the open interval $(-4,5)$ at which g attains a relative minimum. Explain your answer.

d. Find all values of x in the open interval $(-4,5)$ a which the graph of g has a point of inflection.

4. Consider the differential equation $\frac{dy}{dx} = -\frac{3x^3}{y^3}$.

a. Using the axes given, sketch a slope field for the given differential equation at the twelve points indicated.



b. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. Write an equation for the line tangent to the graph at $(1,2)$ and use it to approximate $f(1.1)$.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 2$.

x	1	2	5	7	8	12
f(x)	2	0	-2	3	7	5

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $1 \leq x \leq 12$.

a. Estimate $f'(6)$. Show your work.

b. Evaluate $\int_{1}^{12} 2f'(x) - 5\ dx$. Show your work.

c. Use the left Riemann sum with subintervals indicated by the table to approximate $\int_{1}^{12} f(x)\ dx$.

d. Suppose $f'(5) = 2$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 7$. Use the line tangent to the graph of f at $x = 5$ to show that $f(6) \leq 0$.

6. The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{2n-1}}{3^n \cdot (2n-1)}$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

a. Find the value of R.

b. Write the first four nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 1$.

c. Write the first four nonzero terms of the Maclaurin series for $\ln(1+x)$ about $x = 1$. Use the Maclaurin series for $\ln(1+x)$ about $x = 1$ to write the third-degree Taylor polynomial for $g(x) = \ln(1+x)f(x)$ about $x = 1$.

STOP

END OF EXAM